Friday, March 18th (at 5.00pm, UK time)

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Title: Classical Yang-Baxter equation, Lagrangian multiforms and ultralocal integrable hierarchies.

ABSTRACT

The notion of integrability for classical (field) theories has been almost entirely studied from the Hamiltonian point of view since the early days of the modern theory of integrable systems.

In 2009, the notion of Lagrangian multiform was first put forward by Lobb and Nijhoff as a purely Lagrangian framework to capture integrability. The main idea is to formulate a generalised variational principle for an action involving a certain differential form whose coefficients are interpreted as Lagrangians for a hierarchy. Since its proposal, this idea has flourished in various directions and I will review the main developments for classical field theories in 1+1 dimensions.

Two key ingredients are the multiform Euler-Lagrange equations and the so-called closure relation, both of which derive from the generalised variational principle. One of the most recent developments is the connection of this theory to the well-established theory of the classical r-matrix which had a purely Hamiltonian interpretation so far. I will focus on this aspect in this talk, showing that the classical Yang-Baxter equation underpins the fundamental properties of a certain Lagrangian multiform which contains a large variety of known hierarchies as special cases, such as the Ablowitz-Kaup-Newell-Segur hierarchy, the sine-Gordon (sG) hierarchy and various hierachies related to Zakharov-Mikhailov type models which contain the Faddeev-Reshetikhin (FR) model and recently introduced deformed sigma/Gross-Neveu models as particular cases.

Time permitting, I will also illustrate the versatility of our method by showing how to couple integrable hierarchies together to create new examples of integrable field theories and their hierarchies. We provide two examples: the coupling of the nonlinear Schrödinger system to the FR model and the coupling of sG with the anisotropic FR model.

The most recent results are based on joint work with M. Stoppato and B. Vicedo.
