

Friday, April 21st (at 4.00pm, UK time)

Peter Olver (University of Minnesota)

Title: Two New Developments for Noether's Two Theorems.

ABSTRACT

Noether's First Theorem relates strictly invariant variational problems and conservation laws of their Euler-Lagrange equations. The Noether correspondence was extended by her student Bessel-Hagen to divergence invariant variational problems. In the first part of this talk, I highlight the role of Lie algebra cohomology in the classification of the latter, and conclude with some provocative remarks on the role of invariant variational problems in fundamental physics.

In the second part, I start by recalling the two well-known classes of partial differential equations that admit infinite hierarchies of higher order generalized symmetries:

1. linear and linearizable systems that admit a nontrivial point symmetry group;
2. integrable nonlinear equations such as Korteweg--de Vries, nonlinear Schrödinger, and Burgers'.

I will then introduce a new general class:

3. underdetermined systems of partial differential equations that admit an infinite-dimensional symmetry algebra depending on one or more arbitrary functions of the independent variables

An important subclass of the latter are the underdetermined Euler-Lagrange equations arising from a variational principle that admits an infinite-dimensional variational symmetry algebra depending on one or more arbitrary functions of the independent variables. According to Noether's Second Theorem, the associated Euler-Lagrange equations satisfy Noether dependencies; examples include general relativity, electromagnetism, and parameter-independent variational principles.
