

Generalised higher order Freud weights

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Abstract: Starting with the work of Géza Freud in the fourth quarter of the 20th century, the study of orthogonal polynomials with respect to exponential weights on the real line has flourished. In this talk I will present some recent results on polynomials orthogonal with respect to Freud-type weights, in particular quartic, sextic and higher order Freud weights. I will show that the sequence of generalised higher order Freud weights forms a hierarchy of weights, with associated hierarchies for the first moment that can be written as a finite partition sum of generalised hypergeometric functions. The coefficients in the three-term recurrence relation satisfy difference equations which are members of the first discrete Painlevé hierarchy that can be used to analyse the asymptotic behaviour of the recurrence coefficients. The asymptotic distribution of the zeros of generalised higher order Freud polynomials orthogonal as $n \rightarrow \infty$ follows from the property of regular variation with an appropriate scaling while a mixed recurrence relation satisfied by higher order generalised Freud polynomials is useful in studying other properties of the zeros. Collaborators: Peter Clarkson and Ana Loureiro (University of Kent, UK).