Friday, October 20th (at 4.00pm, UK time)

Jenya Ferapontov (Loughborough)

Title: Differential equations for modular forms and Jacobi forms.

ABSTRACT

It is well known that every modular form f on a discrete subgroup of SL(2, R) satisfies a third-order nonlinear ODE that expresses algebraic dependence of the functions f, f′, f′′, f′′′. These ODEs are automatically invariant under the Lie group SL(2, R), which acts on the solution spaces thereof with an open orbit (and a discrete stabiliser of the generic solution). Similarly, every modular form satisfies a fourth-order nonlinear ODE that is invariant under the Lie group GL(2, R) acting on its solution space with an open orbit. ODEs for modular forms can be compactly expressed in terms of differential invariants of these actions. The invariant forms of both ODEs define plane algebraic curves naturally associated with every modular form; the corresponding ODEs can be seen as modular parametrisations of the associated curves.

After reviewing examples of nonlinear ODEs satisfied by classical modular forms (such as Eisenstein series, modular forms on congruence subgroups of level two and three, theta constants, and some newforms of weight two), we generalise these results to Jacobi forms; these satisfy involutive third-order PDE systems that are invariant under the semidirect product of SL(2, R) with the Heisenberg group.

The talk is based on joint work with Stanislav Opanasenko:

S. Opanasenko, E.V. Ferapontov,

Defining differential equations for modular forms and Jacobi forms;

arXiv:2212.01413.
